A. Proof that $L$ is concave and positive.

We will use the notations previously introduced as well as:

$$z_p = U_n x_p.$$ 

As $L(W) = \sum_p \ell(W, z_p)$, we will simply study $\forall z$, 

$$W \rightarrow \ell(W, z) = \| (I - A) z \|^2 - \| A |W|z| \|^2.$$ 

Observe first that if $\| \{W, A\} \| \leq 1$, then $\| A \| \leq 1$, and:

$$\| Wz \| \leq \| (I - A) z \|$$

Thus, 

$$\| |Wz| | \leq \| (I - A) z \|$$

and:

$$\| A |Wz| | \leq \| (I - A) z \|.$$ 

Consequently, $\ell(W, z) \geq 0, \forall z, \forall W \in \mathcal{C}$. Furthermore, let $W_1, W_2 \in \mathcal{C}$ two operators and $0 \leq \lambda \leq 1$. Then:

$$| (\lambda W_1 + (1 - \lambda) W_2) z | \leq \lambda |W_1| z + (1 - \lambda) |W_2| z$$

where for $x \in \mathbb{R}^n$, $x \geq 0$ iff $x_i \geq 0$. If $Ax > 0$ when $x > 0$, then:

$$A |(\lambda W_1 + (1 - \lambda) W_2) z| \leq \lambda A |W_1| z + (1 - \lambda) A |W_2| z,$$

which implies (as all coordinates are non negative):

$$\| A |(\lambda W_1 + (1 - \lambda) W_2) z| \| \leq \| \lambda A |W_1| z + (1 - \lambda) A |W_2| z| \|,$$

yet one can use the fact that $z \rightarrow ||z||^2$ is convex to conclude. Thus, $W \rightarrow \ell(W, z)$ is convex in $W$.

B. Proof of Proposition 3.5

Proof. Observe that $\mathcal{F}$ linearly conjugates $\mathcal{C}$ to $\{ \hat{W} \in \mathbb{C}^{(2d+1) \times K}, \sum_{k=1}^K |\hat{W}^k[i]|^2 + |W^k[2d+1 - i]|^2 + |A[i]|^2 + |A[2d + 1 - i]|^2 \leq 1, \forall i \leq d, \sum_{k=1}^K |W^k[2d+1]|^2 + |\hat{A}[2d + 1]|^2 \leq 1 \}$. The extremal points of the latter are simply $\mathcal{S}' = \{ \hat{W} \in \mathbb{C}^{(2d+1) \times K}, \sum_{k=1}^K |\hat{W}^k[i]|^2 + |W^k[2d+1 - i]|^2 + |A[i]|^2 + |A[2d + 1 - i]|^2 = 1, \forall i \leq d, \sum_{k=1}^K |W^k[2d+1]|^2 + |\hat{A}[2d + 1]|^2 = 1 \}$, which is conjugated by $\mathcal{F}^+$ to $\mathcal{S}$. But $\mathcal{S}'$ corresponds to the spectrum of an isometry, leading to the conclusion. \(\square\)